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LETTERS TO THE EDITOR

A CENTRIFUGAL PENDULUM ABSORBER FOR ROTATING, HOLLOW ENGINE BLADES

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1. INTRODUCTION

Vibration in turbomachinery has been studied since Campbell [1] first published his pioneering work. Despite over 70 years of research, vibration problems still plague the community. These problems have been attributed to high cycle fatigue (HCF) damage in engine components. The problems can occur on many different engine components, but usually blades are considered the most seriously affected by vibration problems [2]. The vibration forcing function is usually assumed to be harmonic. The excitation frequency is in many instances an integer multiple of the engine speed as represented by the speed lines on a Campbell diagram (see Figure 1). Resonant vibratory response is possible where a speed line crosses a resonant frequency line.

Damping can be added to the blade to reduce the resonant vibration levels. The most common practice is to add friction dampers [3, 4]. Friction damping is often difficult to characterize and can deteriorate with wear. Other damping research for engine blades has focused on the use of viscoelastic material. The major challenges with viscoelastic treatments are the availability of materials that function in the elevated temperature region and the tendency of the material to creep when subjected to the centrifugal force field. Vibration absorbers and particle dampers have also been studied. Jones [5] showed that a damped cantilevered beam located on the tip of the blade could effectively suppress vibration for a particular mode. Other promising work is in the area of particle damping [6].

In the past, the approach has been to add damping to a mode to attenuate resonant response at speed line crossings. This paper presents an alternative view. Rather than adding damping to a mode or family of modes, a damper absorber is constructed that tunes itself to the vibration excitation frequency associated with engine speed. The device can thereby attenuate all resonant responses, as they are excited. The hollow blade damping device is an internal pendulum vibration absorber which exploits artificial gravitational (centrifugal) loading to automatically vary its resonant (absorbing) frequency. Any frequency of vibratory excitation associated with a particular speed line can be tracked with proper selection of the pendulum's moments of inertia. The work is different from previous research on pendulum absorbers [7] which was intended for a constant

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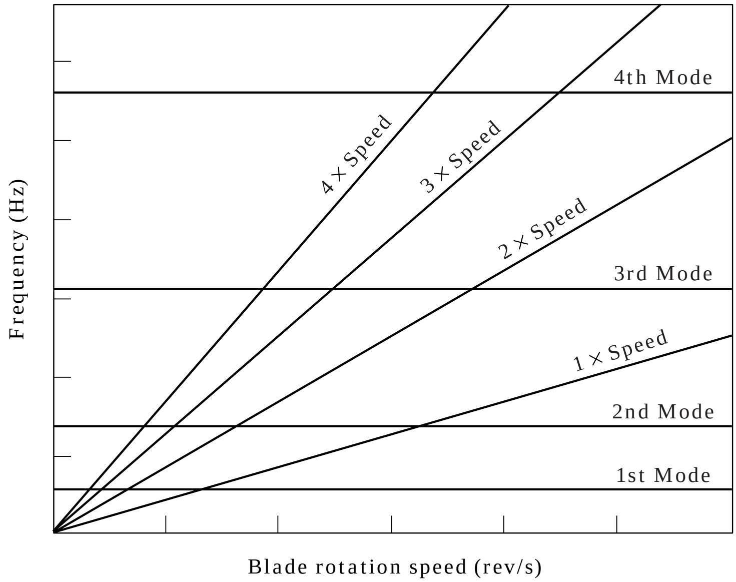

Figure 1. A typical Campbell diagram.

speed application and suppression of a single mode. Here the utility of the pendulum absorber for variable speeds and multiple modes is emphasized. The theoretical development for this device follows.

2. BACKGROUND

Our damping device is in fact a tuned mass damper. This class of devices is also referred to as damped vibration absorbers, vibration absorbers, or proof-mass-dampers. Here for brevity, they will be referred to as absorbers. They come in many forms including mechanical, piezoelectric, magnetic, and acoustic. No matter the form, the absorber adds an additional degree-of-freedom to a system. The additional degree-of-freedom is tuned to a system resonant frequency. If the absorber is undamped, the absorber simply attenuates the original resonant response, replacing it with a localized resonant response within the absorber. For a pure harmonic excitation of a particular resonance, this may be adequate. For a broader band excitation, a damped absorber is usually preferred. The damped absorber provides reduced response of the absorber degree-of-freedom as well as attenuation of the system resonance. In addition, damped absorbers have broader bandwidths making them more tolerant to mistuning than undamped absorbers.

Rather than tuning an absorber to a particular mode, we will use centrifugal loading to tune the absorber to the component excitation frequency. Speed-tracking centrifugal pendulums are well known in the literature. They have been used in the past to absorb torsional vibrations in shafts [8, 9] and for linear vibration of rotating machinery [10]. Here the centrifugal pendulum is applied to the engine blade where the vibration problem is considerably more complex. The tuning properties depend on the pendulum moments of inertia and the blade orientation. Our centrifugal pendulum will be damped to provide tolerance to mistuning.

3. THEORY AND DISCUSSION

The analysis of a pendulum absorber for a rotating, vibrating blade can be complicated. The derivation is presented in this section in a series of steps. First the effects of an undamped pendulum on a single-degree-of-freedom system will be derived. This is a simple problem which demonstrates the basis of the overall problem. Next, the effects of centrifugal loading on pendulum resonant frequencies will be derived. Finally a general formulation with a damped pendulum applied to a damped resonance will be presented. This formulation will allow the effects of mistuning to be investigated.

3.1. *The undamped pendulum absorber*

Figure 2 shows a pendulum attached to a spring–mass system. The spring–mass system represents a vibration mode of an engine blade. The mass of the blade is represented by m_1, and the mass of the pendulum is represented by m_2. The blade mass is excited by a harmonic force with a frequency which varies linearly with blade rotation rate. Without the pendulum absorber, the mass would resonant when the engine produces a harmonic force at the natural frequency of the spring–mass system.

The effects of the pendulum absorber are determined by analyzing the equations of motion (EOM), which are

$$(m_1 + m_2)\ddot{x} + m_2 L\ddot{\theta}\cos\theta + kx - m_2 L\dot{\theta}^2 \sin\theta = F(t), \tag{1}$$

$$\ddot{x}\cos\theta + L\ddot{\theta} + g\sin\theta - \dot{x}\dot{\theta}\sin\theta = 0. \tag{2}$$

These are linearized and Fourier transformed to produce

$$\begin{bmatrix} -(m_1+m_2)\omega^2 + k & -m_2 L\omega^2 \\ -\omega^2/L & -\omega^2 + g/L \end{bmatrix} \begin{Bmatrix} X(\mathrm{j}\omega) \\ \theta(\mathrm{j}\omega) \end{Bmatrix} = \begin{Bmatrix} F(\mathrm{j}\omega) \\ 0 \end{Bmatrix}. \tag{3}$$

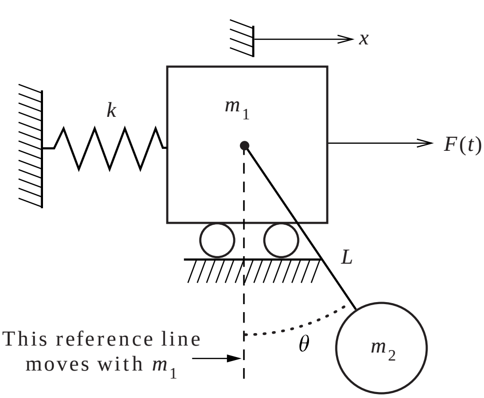

Figure 2. A spring–mass system representing a blade mode. Attached to the mass is an undamped pendulum absorber.

The resulting transfer function between the blade mass displacement and the forcing function is

$$\frac{X}{(F/m_1)} = \frac{\omega_p^2 - \omega^2}{[\omega_b^2 - (1+\beta)\omega^2](\omega_p^2 - \omega^2) - \beta\omega^4}, \tag{4}$$

where

$$\omega_b^2 = k/m_1, \qquad \omega_p^2 = g/L, \qquad \beta = m_2/m_1. \tag{5}$$

The natural frequency of the spring–mass system (representing the blade) when disconnected from the pendulum is ω_b. The natural frequency of the pendulum when disconnected from the mass is ω_p. The mass ratio is defined as β.

According to equation (4), the vibration of the mass can be cancelled if the natural frequency of the pendulum exactly equals the forcing frequency. This is true regardless of the natural frequency of the original spring–mass system. Thus if the pendulum natural frequency can track along the speed line, the vibrations for all modes excited by a speed line can be cancelled. In a normal gravitational field, this is impossible.

3.2. *The centrifugal pendulums*

The natural frequency of the pendulum absorber can track a speed line in a centrifugal force field. Shown in Figure 3 is the geometry for this case. The pendulum absorber is represented by a rigid plate located inside a flat hollow

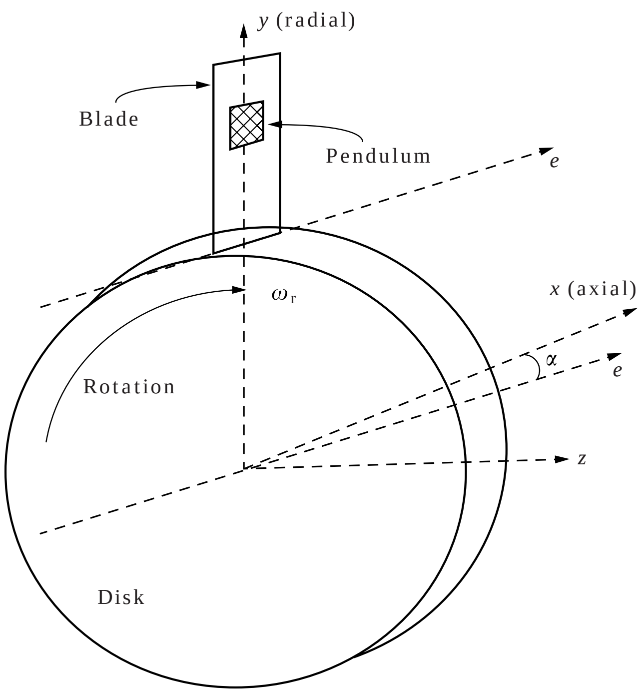

Figure 3. A pendulum absorber inside a hollow engine blade. The blade is attached to a rotating disk at an angle. The pendulum rotates out of the plane of the blade. Note: the coordinate system rotates about the x-axis (the same for Figures 4, 5 and 6).

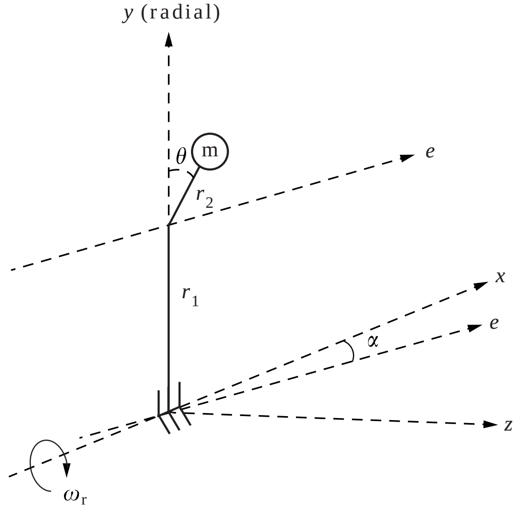

Figure 4. A simple model of the centrifugal pendulum absorber. The blade and disk are represented by a rotating linkage. The pendulum is idealized as a point mass.

blade. The blade is rigidly attached to a rotating disk. The plate is pinned so that it can rotate about an axis parallel to the line of attachment of the blade to the disk. The angle between the disk's axis of rotation and the plane of the blade is defined as α. The natural frequency of the pendulum will depend upon the blade angle, α, the rotational speed of the disk, ω_r, and mass distribution of the pendulum. In this section, the expression for the natural frequency will be derived.

The complicated geometry of the blade–absorber is simplified in Figure 4. The blade and disk are represented by a rotating link. For the moment, the link is assumed to be rigid. The length of the link, r_1, is the distance from the disk's axis of rotation to the pendulum axis of rotation. The angle between the axes of rotation is the blade angle, α. The pendulum is idealized initially as a point mass. The angle between the blade and the pendulum is θ. The blade and disk are rotating about the x-axis at the radial speed of ω_r. The coordinate system also rotates about the x-axis at this speed (i.e., the y–z plane rotates about the x-axis).

An expression for the kinetic energy of the pendulum is needed to determine the natural frequency of the pendulum. The kinetic energy in terms of the time derivative of the mass position vector, $\mathbf{r}$, is

$$T = 1/2m(\dot{\mathbf{r}}\cdot\dot{\mathbf{r}}). \tag{6}$$

The position vector is

$$\mathbf{r} = r_1\mathbf{e}_{r1} + r_2\mathbf{e}_{r2}, \tag{7}$$

where $\mathbf{e}_{r1}$ and $\mathbf{e}_{r2}$ are unit vectors defining the orientation of the two links. These unit vectors are

$$\mathbf{e}_{r1} = \mathbf{j}, \qquad \mathbf{e}_{r2} = \cos\theta\mathbf{j} + \sin\theta\,(\cos\alpha\mathbf{k} - \sin\alpha\mathbf{i}), \tag{8, 9}$$

where **i**, **j**, and **k** are unit vectors in the x, y, and z directions. The time derivative of the position vector is

$$\dot{r} = r_1 \dot{\mathbf{e}}_{r1} + r_2 \dot{\mathbf{e}}_{r2}, \tag{10}$$

which in terms of cross products is

$$\dot{r} = r_1\{\omega_r \mathbf{i} X \mathbf{e}_{r1}\} + r_2\{[\omega_r \mathbf{i} + \dot{\theta}(\cos\alpha \mathbf{i} + \sin\alpha \mathbf{k})] X \mathbf{e}_{r2}\}. \tag{11}$$

After simplification, the kinetic energy becomes

$$T = 1/2m[\omega_{\mathrm{r}}^2(r_1 + r_2\cos\theta)^2 + r_2^2\dot{\theta}^2 + \omega_{\mathrm{r}}^2 \mathrm{r}_2^2 \sin^2\theta\cos^2\alpha$$
$$+ 2\omega_{\mathrm{r}}\dot{\theta}\cos\alpha(r_2^2 + r_1 r_2\cos\theta)]. \tag{12}$$

The potential energy term due to the gravitational effect is neglected in this analysis. Lagrange's equations of motion are used to generate the EOM, which after linearization is

$$\ddot{\theta} + \omega_{\mathrm{r}}^2(\sin^2\alpha + (r_1/r_2))\theta = 0. \tag{13}$$

The natural frequency of the pendulum is easily extracted from the EOM as

$$\omega_{\mathrm{p}} = \omega_{\mathrm{r}}\sqrt{\sin^2\alpha + (r_1/r_2)}. \tag{14}$$

The pendulum tracks a speed line if the radical of equation (14) equals the order of the speed line. If the blade angle is fixed, there are two variable design parameters. The range of one parameter, the distance r_1, is limited by the disk radius and the blade length. The range of the other, the pendulum arm length, r_2, is limited by the available space inside the blade. Therefore, it may not be possible to design a lumped pendulum to track a desired speed line. Fortunately all practical pendulums have distributed mass. The mass can be distributed to track a given speed line despite the space limitations.

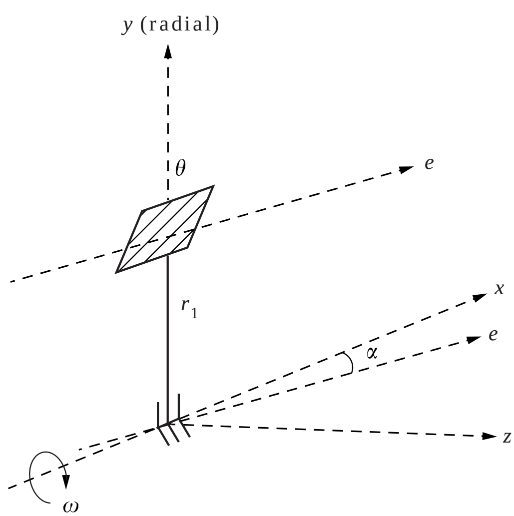

Figure 5. A simple model of the distributed centrifugal pendulum absorber.

The simplified geometry for the distributed pendulum is shown in Figure 5. The pendulum is represented by a rigid plate rotating about an axis. The plate has constant thickness and a constant width. The plate length above the rotation axis is L_2 and the length below the axis is L_1. The kinetic energy term for the distributed pendulum is

$$T = 1/2\rho A \int_{L_1}^{L_2} [\omega_r^2(r_1 + r_2\cos\theta)^2 + r_2^2\dot{\theta}^2 + \omega_r^2 r_2^2 \sin^2\theta\cos^2\alpha + 2\omega_r\dot{\theta}\cos\alpha(r_2^2 + r_1 r_2\cos\theta)]\mathrm{d}r_2, \tag{15}$$

where A is the cross-sectional area and ρ is the mass density of the pendulum plate. After integrating, applying Lagrange's equations of motion, and linearization, the natural frequency of the distributed pendulum is found to be

$$\omega_p = \omega_r\sqrt{\sin^2\alpha + r_1(I_1/I_2)}, \tag{16}$$

where I_1 and I_2 are the first and second moments of area of the pendulum plate

$$I_1 = \int_{L_1}^{L_2} r_2\mathrm{d}r_2 = 1/2(L_2^2 - L_1^2), \qquad I_2 = \int_{L_1}^{L_2} r_2^2\mathrm{d}r_2 = 1/3(L_2^3 + L_1^3). \quad (17, 18)$$

The area distribution of the pendulum can be adjusted to track a particular speed line. The shape and thickness of the pendulum can also be varied. The expression for the natural frequency for this case is given by equation (16).

There are two special cases of the centrifugal pendulum absorber when α is 0° or 90°. Figure 6 shows the simplified geometry for these cases. When α is 0°, the pendulum oscillates in plane with the disk–blade rotation. This geometry is

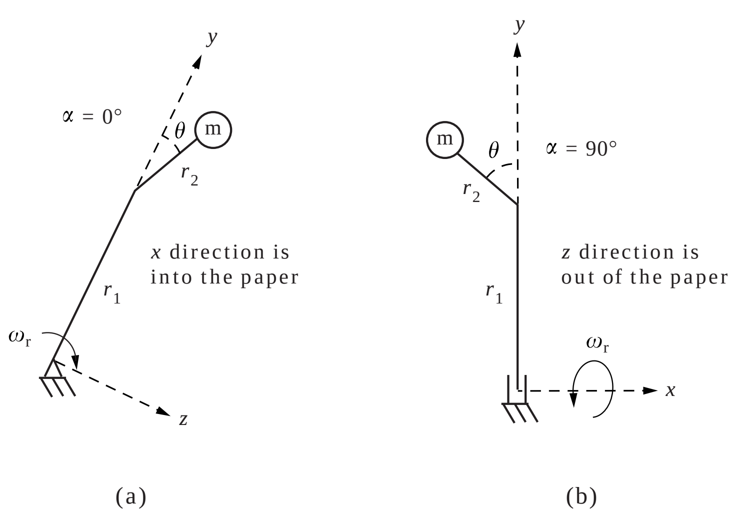

Figure 6. Two special cases of the centrifugal absorber: (a) the pendulum rotates in the same plane as the rotating disk ($\alpha = 0°$), and (b) the pendulum rotates completely out of the plane of the disk ($\alpha = 90°$).

identical to the centrifugal pendulum absorber used for torsion vibration in shafts [8, 9]. When the blade angle, α, is 90°, the pendulum oscillates entirely out of the rotational plane. This geometry is identical to the centrifugal pendulum absorbers for linear motion [10]. Again, the pendulum for each case can be distributed.

3.3. *The damped pendulum absorber*

The undamped pendulum absorber cancels response due to a harmonic force if the absorber is exactly tuned to the excitation frequency. That is, if the pendulum mass can be distributed properly and the excitation is purely harmonic. However there will be limits in the precision of the pendulum. The excitation may not be purely harmonic, perhaps being narrow banded. A damped absorber is more robust for these cases. The engineering tradeoff is that the vibration will be attenuated rather than cancelled. But an added benefit is that the pendulum itself will be damped and less prone to fatigue. In this section, the transfer function for a damped absorber will be derived. Damping in the blade will also be added to the analysis.

Figure 7 shows the configuration for the damped absorber. The linearized, Fourier transformed EOM are

$$\begin{bmatrix} k-(m_1+m_2)\omega^2+\mathrm{j}\omega c_1 & -m_2L\omega^2 \\ -\omega^2/L & \omega_\mathrm{p}^2-\omega^2+\mathrm{j}\omega c_2/m_2 \end{bmatrix} \begin{Bmatrix} X(\mathrm{j}\omega) \\ \theta(\mathrm{j}\omega) \end{Bmatrix} = \begin{Bmatrix} F(\mathrm{j}\omega) \\ 0 \end{Bmatrix}. \tag{19}$$

In these equations, ω_p is the natural frequency of the pendulum in the centrifugal field. For simplicity, the inherent damping of the blade is represented by the viscous damper c_1. The equations are also simplified by using the definitions of the mass ratio and the original spring–mass resonance

$$\begin{bmatrix} \omega_\mathrm{b}^2-(1+\beta)\omega^2+\mathrm{j}2\zeta_\mathrm{b}\omega_\mathrm{b}\omega & -\beta L\omega^2 \\ -\omega^2/L & \omega_\mathrm{p}^2-\omega^2+\mathrm{j}2\zeta_\mathrm{p}\omega_\mathrm{b}\omega \end{bmatrix} \begin{Bmatrix} X(\mathrm{j}\omega) \\ \theta(\mathrm{j}\omega) \end{Bmatrix} = \begin{Bmatrix} F(\mathrm{j}\omega)/m_1 \\ 0 \end{Bmatrix}, \tag{20}$$

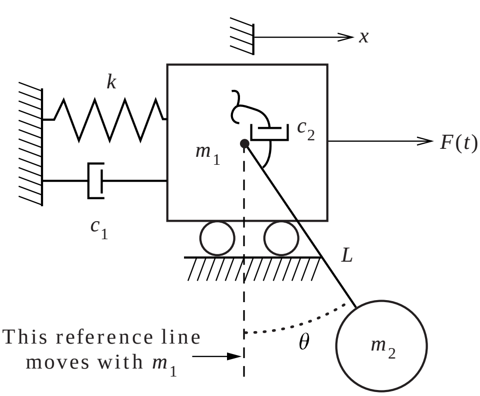

Figure 7. The damped pendulum absorber applied to a damped mode.

where

$$c_1/m_1 = 2\zeta_b\omega_b, \qquad c_2/m_2 = 2\zeta_p\omega_b. \tag{21}$$

The pendulum's damping ratio, ζ_p, is referenced to a fixed frequency, ω_b, not the variable pendulum frequency.

As the pendulum frequency tracks along the engine speed line, there may be some precision error signified by a multiplicative factor

$$\omega_p = h\omega, \tag{22}$$

where h is a number near one, which will referred to as the tuning ratio. Normally the tuning ratio refers to the ratio between the absorber frequency and the system frequency; here it refers to the ratio between the absorber frequency and the excitation frequency. Substituting equation (22) into the EOM produces

$$\begin{bmatrix} 1-(1+\beta)\gamma^2 + \mathrm{j}2\zeta_b\gamma & -\beta L\gamma^2 \\ -\gamma^2/L & \gamma^2(h^2-1)+\mathrm{j}2\zeta_p\gamma \end{bmatrix} \begin{Bmatrix} X(\mathrm{j}\omega) \\ \theta(\mathrm{j}\omega) \end{Bmatrix} = \begin{Bmatrix} X^{ST} \\ 0 \end{Bmatrix}, \tag{23}$$

where the following substitutions are also made:

$$X^{ST} = F/k, \qquad \gamma = \omega/\omega_b. \tag{24}$$

The transfer function between the blade mass displacement and the forcing function (in non-dimensional form) is

$$\frac{X}{X^{ST}} = \frac{\gamma^2(h^2-1)+\mathrm{j}2\zeta_p\gamma}{[\gamma^2(h^2-1)+\mathrm{j}2\zeta_p\gamma][1-(1+\beta)\gamma^2+\mathrm{j}2\zeta_b\gamma]-\beta\gamma^4}. \tag{25}$$

The effects of the absorber can not be determined as clearly as with equation (4). Comparison can be made to the response of the baseline system (i.e., no absorber), whose transfer function is

$$\frac{X}{X^{ST}} = \frac{1}{1-\gamma^2+\mathrm{j}2\zeta_b\gamma}. \tag{26}$$

There are various measures to quantify the effects of the absorber; one is shown in Figure 8. Here the reduction in peak response for various tuning ratios and pendulum damping ratios (for $\beta = 0{\cdot}01$ and $\zeta_b = 0{\cdot}25\%$) is plotted. Notice that for an undamped absorber ($\zeta_p = 0$), small amounts of mistuning severely degrade the absorber performance. It is primarily for this reason that some amount of absorber damping is necessary.

3.4. *Practical concerns*

The centrifugal absorber can be an effective method of vibration suppression, but there are many concerns about implementation. The first of which is the practical design of the absorber. A hollow blade is required unless the absorber can be mounted on the tip of the blade (which creates other problems).

The mechanism for the pendulum damping has not been specified. Some aerodynamic damping could exist inside the hollow blade. As discussed before, damping provides a more robust absorber, but it also attenuates the response of

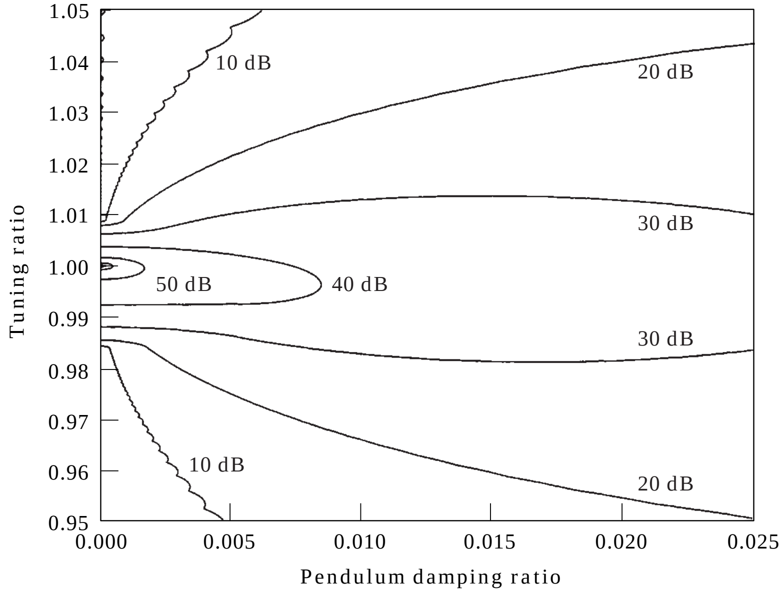

Figure 8. The reduction in peak response provided by the damped centrifugal absorber over the baseline system. For this plot, the mass ratio is 0·01 and the blade viscous damping is 0·25% of critical.

the absorber itself and ameliorates some of the wear due to oscillation. Wear will eventually detune the absorber and possibly result in failure of the pendulum. If the failure frees the pendulum from its axis of rotation, a rotating imbalance may cause even greater problems.

Another limitation is that a single absorber can track only one speed line. For a given operating range of the engine, there may be many speed lines that excite the blade. For each of these speed lines, an absorber may be necessary.

Location selection has not been addressed in the preceding derivations. Presumably there are better locations than others to effect certain modes. The orientation of the pendulum relative to the vibration mode has also not been addressed. The pendulum axis is aligned with the blade bending axis, which is ideal for low order bending modes. The equations for the pendulum natural frequency can accommodate other orientations. However higher order modes may have motion in multiple directions. Nowhere in the derivations are these observability/controllability issues addressed.

4. SUMMARY

The analytical derivation of a speed tracking pendulum absorber has been presented. The intended application of the absorber is engine blade vibration suppression. The derivation shows it is possible to use centrifugal loading to automatically tune the absorber's frequency to an engine speed line, so that the absorber will suppress vibrations excited by that particular speed line. The mass distribution of the pendulum has to be precisely designed for a given blade angle and engine order for the absorber to function properly. The pendulum absorber should also have some inherent damping to prevent minor mistuning from rendering the absorber ineffective. Damping in the absorber also attenuates absorber response. Other practical concerns are also discussed.

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